

# On interpretation of recent proper motion data for the Large Magellanic Cloud

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## ABSTRACT

Recent observational studies using the *Hubble Space Telescope* (*HST*) have derived the center-of-mass proper motion (CMPM) of the Large Magellanic Cloud (LMC). Although these studies carefully treated both rotation and perspective effects in deriving the proper motion for each of the sampled fields, they did not consider the effects of local random motion in the derivation. This means that the average PM of the fields (i.e., the observed CMPM) could significantly deviate from the true CMPM, because the effect of local random motion can not be close to zero in making the average PM for the small number of the fields ( $\sim 10$ ). We discuss how significantly the observationally derived CMPM can deviate from the true CMPM by applying the same method as used in the observations for a dynamical model of the LMC with a known true CMPM. We find that the deviation can be as large as  $\sim 50 \text{ km s}^{-1}$  ( $\sim 0.21 \text{ mas yr}^{-1}$ ), if the LMC has a thick disk and a maximum circular velocity of  $\sim 120 \text{ km s}^{-1}$ . We also find that the deviation depends both on the total number of the sampled fields and on structure and kinematics of the LMC. We therefore suggest that there is a possibility that the observed CMPM of the LMC deviates from the true one to some extent. We also show that a simple mean of PM for a large number of the LMC fields ( $\sim 1000$ ) can be much closer to the true CMPM.

*Subject headings:* Magellanic Clouds

## 1. Introduction

The orbital evolution of the Magellanic Clouds (MCs) with respect to the Galaxy has been considered to be one of key parameters that control dynamical and chemical evolution of the LMC and the Small Magellanic Cloud (SMC) and the formation processes of the Magellanic Stream (MS) and its leading arms (e.g., Murai & Fujimoto 1980, MF80; Gardiner

& Noguchi 1996, GN96; Bekki & Chiba 2005, BC05; Mastropietro et al. 2005). Recent observational studies on the CMPM for the LMC using the High Resolution Camera (HRC) of the Advanced Camera for Surveys (ACS) on the *HST* have derived an accurate present 3D motion of the LMC around the Galaxy (e.g., Kallivayalil et al. 2006, K06a; Piatek et al. 2008, P08). One of remarkable results from these studies is that the LMC can possibly have a velocity with respect to the Galaxy ( $V_{\text{LMC}}$ ) being  $\sim 380 \text{ km s}^{-1}$  (Kallivayalil et al. 2006), which is significantly larger than that ( $V_{\text{LMC}} \sim 300 \text{ km s}^{-1}$ ) predicted by one of promising theoretical models for the formation of the MS (e.g., GN96). The observational results thus have profound implications on the past orbits of the MCs (e.g., Besla et al. 2007; Kallivayalil et al. 2006, K06b), a possible common halo of the MCs (Bekki 2008), and the formation processes of the MS (e.g., Diaz & Bekki 2011, DB11; Ruzicka et al. 2010, R10).

The previous work by K06a considered that the observed PM for each field (PM(field)) is a combination of the PM of the center-of-mass (CM) for the LMC (PM(CM)) and the field-dependent residual (PM<sub>res</sub>(field)) as follows:

$$\text{PM}(\text{field}) = \text{PM}(\text{CM}) + \text{PM}_{\text{res}}(\text{field}). \quad (1)$$

In estimating the PM of the LMC CM (i.e., PM<sub>est</sub>(CM)) for each of the selected high-quality 13 fields, K06a very carefully considered how the internal rotation of the LMC (“rotation effect”) and the viewing angle (“perspective effect”) influence PM<sub>res</sub>(field) and thereby made an average of the 13 PM<sub>est</sub>(CM) to derive the CMPM. Since the average PM is not a *simple* average of the observed PM of the 13 fields (i.e., not the average of PM(field)), the observed CMPM can be pretty close to the true one, if all stars have almost circular motion and if the LMC has a very thin disk. However, the LMC has a thick disk with a bar (e.g., van der Marel et al. 2002, vdM02), which is indicative of larger local random motion both in radial and vertical directions (i.e., deviation from circular motion). Therefore, PM<sub>est</sub>(CM) for each field can significantly deviate from the true PM of the LMC and the average PM<sub>est</sub>(CM) can also deviate from the true CMPM if the number of the sampled field is small.

The purpose of this Letter is to show how significantly the observationally derived CMPM can deviate from the true one by using a dynamical (N-body) model for the LMC with a known true CMPM. In the present study, we first pick up randomly stellar particles with the particle number ( $N$ ) of 3 – 3000 in a N-body model for the LMC with a given structure and kinematics and thereby derive the CMPM of the LMC in the same way as done in previous observational studies. We then compare the derived CMPM with the true one so that we can discuss the possible difference between the two CMPMs. This investigation is quite important, because the possible difference between the observed and true CMPMs can not be observationally discussed owing to the lack of detailed information of the 3D positions and velocities of stars in each field.

Recent different observational studies on the CMPM of the LMC have revealed different CMPM and maximum circular velocity ( $V_c$ ) of the LMC (e.g., K06a, P08, and Costa et al. 2009; C09), and different PM studies using almost the same data set and adopting a similar PM measurement method have derived different CMPMs and  $V_c$  of the LMC (K06a and P08): this is yet to be explained. Furthermore P08 already suggested that a significant scatter ( $\sim 30 \text{ km s}^{-1}$ ) in the derived PM of the sampled 21 LMC fields is due to significant departures from circular motion. Thus it is crucial to investigate how random motion in the LMC can affect the observational estimation of the CMPM in a quantitative way.

## 2. The model

### 2.1. The LMC

The present LMC model is consistent with a high-mass model in BC05 in terms of the disk structure and the dark matter density profile, but it is slightly different from BC05 in the dark matter fraction and the inner rotation curve profile (also the absence of gas). The modeled LMC is consistent with the observed radial structure of the disk (e.g., Bothun & Thompson 1988), the total mass (e.g., Westerlund 1997; P08), structure and kinematics of the thick disk (vdM02), and dark matter content (vdM02). The LMC is composed of a dark matter halo and a stellar disk with the total masses being  $M_{\text{dm}}$  and  $M_{\text{d}}$ , respectively. Following the observational results by vdM02 showing  $M_{\text{dm}} = (8.7 \pm 4.3) \times 10^9 M_{\odot}$  within 9kpc of the LMC, we assume that a reasonable mass fraction of the dark matter halo ( $f_{\text{dm}} = M_{\text{dm}}/(M_{\text{dm}} + M_{\text{d}})$ ) is 0.50-0.67 within the adopted LMC size. We adopted an NFW halo density distribution (Navarro, Frenk & White 1996) suggested from CDM simulations and the “c”-parameter is set to be 12. The dark matter halo is truncated at the observationally suggested tidal radius of the LMC ( $\sim 15 \text{ kpc}$ ; vdM02). We mainly investigate the “fiducial” LMC model with the total mass ( $M_{\text{t}} = M_{\text{dm}} + M_{\text{d}}$ ) of  $2 \times 10^{10} M_{\odot}$ ,  $f_{\text{dm}} = 0.5$ , and  $V_c = 117 \text{ km s}^{-1}$ .

The radial ( $R$ ) and vertical ( $Z$ ) density profiles of the disk (with the size  $R_{\text{d}}$  of 7.5 kpc) were assumed to be proportional to  $\exp(-R/R_{d,0})$ , with scale length  $R_{d,0} = 0.2R_{\text{d}}$ , and  $\text{sech}^2(Z/Z_{d,0})$ , with scale length  $Z_{d,0} = 0.06R_{\text{d}}$ , respectively: the stellar disk has the radial and vertical scale length of 1.5 kpc and 0.45 kpc, respectively. In addition to the rotational velocity caused both by the gravitational fields of the disk and dark halo components, the initial radial and azimuthal velocity dispersions were assigned to the disk component according to the epicyclic theory with Toomre’s parameter  $Q$  (Binney & Tremaine 1987) ranging from 0 to 3 in the present study. By investigating models with different  $Q$ , we can discuss how local random motion (due to non-zero velocity dispersions) can introduce differences

between the observed and true CMPMs. We run the LMC disk model for 20 dynamical time scales so that we can construct a “barred model” for our investigation. There is no significant differences in stellar kinematics between the above relaxed and unrelaxed models. The line-of-sight velocity dispersions in the  $x$ -,  $y$ -, and  $z$ -components of stellar velocities ( $\sigma_x$ ,  $\sigma_y$ , and  $\sigma_z$ , respectively) reach their maximum values of  $55 \text{ km s}^{-1}$ ,  $55 \text{ km s}^{-1}$ , and  $43 \text{ km s}^{-1}$ , respectively, at  $R = 0$  in the standard LMC model with  $Q = 1.5$ .

## 2.2. Estimation of possible differences between simulated and true center-of-mass velocities

We investigate possible differences between observed and true 3D velocities of the CM of the LMC rather than the CMPM differences, because we can more clearly show the differences without considering the coordinate transformation and the present location of the LMC with respect to the Galaxy. The CM of the LMC is assumed to be located at the center of the coordinate (i.e.,  $(x, y, z) = (0, 0, 0)$ ) in all models. The spin of the LMC disk is specified by two angles  $\theta$  and  $\phi$  in units of degrees, where  $\theta$  is the angle between the  $z$ -axis and the vector of the angular momentum of the disk, and  $\phi$  is the azimuthal angle measured from  $x$  axis to the projection of the angular momentum vector of the disk onto the  $x$ - $y$  plane.

The LMC in a model is assumed to be moving with a “true” 3D CM velocity of  $\mathbf{V} = (V_x, V_y, V_z)$  and therefore the initial velocity of each  $i$ -th stellar particle ( $\mathbf{v}_i$ ) within the LMC is described as:

$$\mathbf{v}_i = \mathbf{V} + \mathbf{V}_i, \quad (2)$$

where  $\mathbf{V}_i$  is the 3D motion of the particle with respect to the center of mass of the LMC and depends on the location of the particle and the adopted dynamical model of the LMC (e.g.,  $V_c$ ).

We first randomly pick up stellar particles with the total number of  $N$  from 200000 particles in the LMC model. Each selected stellar particle is not literally a star but represents a field with local kinematics. Then we derive the 3D velocity of the CM ( $\mathbf{v}_{\text{cm},i}$ ) from that of each field as follows:

$$\mathbf{v}_{\text{cm},i} = \mathbf{v}_i - \mathbf{V}_{c,i}, \quad (3)$$

where  $\mathbf{V}_{c,i}$  is the 3D circular velocity vector at the position of the field and thus depends on inclination angles (i.e.,  $\theta$  and  $\phi$ ) of the LMC and dynamical properties of the LMC (e.g.,  $V_c$ ): thus we here consider both rotation and perspective effects properly. This derivation of  $\mathbf{v}_{\text{cm},i}$  is done in the same way as done in the previous *HST* PM studies (e.g., K06a). Unlike observations, there is no uncertainty in  $\mathbf{V}_{c,i}$ , because both the radial dependence of

the rotation curve and inclination angles (corresponding to viewing angles in observations) in the LMC are precisely considered.

Thus the simulated CM 3D velocity ( $\mathbf{V}_{\text{sim}}$ ) is described as follows:

$$\mathbf{V}_{\text{sim}} = \frac{1}{N} \sum_{i=1}^N \mathbf{v}_{\text{cm},i}. \quad (4)$$

The above  $\mathbf{v}_{\text{cm},i}$  can deviate significantly from the true  $\mathbf{V}$  because of velocity dispersions of the LMC. We estimate the deviation for the sampled  $N$  fields in a quantitative way as follows:

$$\Delta \mathbf{V}_m = \mathbf{V}_{\text{sim}} - \mathbf{V}. \quad (5)$$

We also derive the difference between the simulated and true 3D velocities ( $\Delta \mathbf{V}_i$ ) for each field as follows:

$$\Delta \mathbf{V}_i = \mathbf{v}_{\text{cm},i} - \mathbf{V}. \quad (6)$$

We estimate  $\Delta \mathbf{V}_m$  ( $= (\Delta V_{m,x}, \Delta V_{m,y}, \Delta V_{m,z})$ ) and the absolute magnitude of the projected velocity ( $\Delta V_m = \sqrt{\Delta V_{m,x}^2 + \Delta V_{m,y}^2}$ ) for each sample. The method used for deriving  $\mathbf{V}_{\text{sim}}$  by the above equations (3)-(6) is referred to as “the standard” method.

We also adopt a “simple method” in which  $\mathbf{V}_{\text{sim}}$  is derived without considering rotation and perspective effects: we estimate  $\mathbf{V}_{\text{sim}}$  simply by making an average of  $\mathbf{v}_i$  (i.e., ignoring  $\mathbf{V}_{c,i}$  in the equation (3)). By comparing the results derived by the simple and standard methods, we can demonstrate that it is important to consider rotation and perspective effects in deriving the CPM of the LMC. The present results do not depend on  $\mathbf{V}$  at all, and thus we show the results for the models with  $\mathbf{V}=(0, 0, 0)$  for clarity. We mainly show the results for the standard LMC model with  $\theta = 45^\circ$  and  $\phi = 30^\circ$  in the present study, because the results of other models are essentially the same as those in this standard model.

### 3. Results

Fig. 1 shows that the difference between the velocity of each field and true CM velocity can be quite large and become up to  $\sim 100 \text{ km s}^{-1}$  in each velocity component for some fields, if the simple method (i.e.,  $\mathbf{v}_{\text{cm},i} - \mathbf{V} = \mathbf{v}_i - \mathbf{V}$ ) is applied for this fiducial LMC model with  $V_c = 117 \text{ km s}^{-1}$ . Also the simulated CM velocity for the 13 fields ( $\mathbf{V}_{\text{sim}}$ ) can not be close to the true one owing to the local random motion ( $\Delta V_m \sim 33 \text{ km s}^{-1}$ ). On the other hand, although the difference between the velocity of each field and the true CM velocity can be large (up to  $\sim 50 \text{ km s}^{-1}$ ) in the standard method, the simulated LMC CM velocity ( $\mathbf{V}_{\text{sim}}$ ) can be pretty close to the true one ( $\Delta V_m \sim 8 \text{ km s}^{-1}$ ). This clearly demonstrates

that if rotation and perspective effects are taken into account, the simulated CM velocity can be closer to the true one.

Fig. 2 shows how the deviation from the simulated LMC CM velocity ( $\mathbf{V}_{\text{sim}}$ ) from the true CM one depends on  $Q$  and presence or absence of the central stellar bar in the LMC disk. The simulated CM velocities are derived for 1000 different data samples each of which has  $N$  fields. The difference between the simulated and true velocities can be as large as  $50 \text{ km s}^{-1}$  in the non-barred model with  $Q = 1.5$ , though the average  $\Delta V_{\text{m}}$  for the 1000 samples is only  $\sim 22 \text{ km s}^{-1}$ . The origin of the significant deviation in some samples is due to local random motion in the LMC disk. It is also clear that the simulated CM velocity can deviate more significantly from the true one in the model with a higher  $Q$  owing to the larger degree of local random motion: the average  $\Delta V_{\text{m}}$  is  $\sim 35 \text{ km s}^{-1}$  for the non-barred model with  $Q = 3.0$ . The average  $\Delta V_{\text{m}}$  is  $\sim 25 \text{ km s}^{-1}$  for the barred model, which means that there is no remarkable difference between non-barred and barred models.

Fig. 3 shows the degree of deviation of the simulated LMC CM velocity from the true one depends strongly on the number of fields ( $N$ ) such that it can be higher in models with smaller  $N$ : the average  $\Delta V_{\text{m}}$  for  $N = 3, 30$ , and  $300$  are  $40 \text{ km s}^{-1}$ ,  $12 \text{ km s}^{-1}$ , and  $4 \text{ km s}^{-1}$ , respectively. It should be stressed here that even if 30 fields are used, the deviation can be still as large as  $30 \text{ km s}^{-1}$  in each velocity component. Fig. 3 also shows that the deviation of the simulated CM velocity can be quite small ( $< 10 \text{ km s}^{-1}$ ), if 300 field are used. This result implies that at least  $\sim 300$  fields are necessary to make a very accurate CMPM of the LMC if we estimate the CMPM using the method by K06a. Fig. 4 demonstrates that even if we adopt the simple method, the difference between the simulated and true CM velocities can be well less than  $10 \text{ km s}^{-1}$  for  $N > 300$ . This result implies that the simple average of the observed PMs for a large number of fields (or stars) can give a quite accurate CMPM of the LMC.

The present results depend on model parameters as follows: The deviation of the simulated LMC CM velocity from the true one can be smaller in the low-mass LMC model with  $M_{\text{t}} = 10^{10} M_{\odot}$ ,  $f_{\text{dm}} = 0.5$ , and  $V_{\text{c}} = 83 \text{ km s}^{-1}$ : the mean  $V_{\text{m}}$  is  $16 \text{ km s}^{-1}$ . The present results do not depend strongly on  $f_{\text{dm}}$ : the mean  $V_{\text{m}}$  for  $f_{\text{dm}} = 0.5$  and  $0.67$  are  $22 \text{ km s}^{-1}$  and  $18 \text{ km s}^{-1}$ , respectively. The deviation of the simulated CM velocity of the LMC from the true one can be larger in the models with higher inclination angles; for example, the mean  $V_{\text{m}}$  is  $34 \text{ km s}^{-1}$  for  $\theta = 60^{\circ}$  and  $\phi = 80^{\circ}$ .

#### 4. Discussion and conclusions

The present study has first demonstrated that the LMC CMPM derived from  $\sim 10 - 20$  fields can possibly deviate significantly from the true one (up to  $\Delta V_m \sim 50 \text{ km s}^{-1}$ ) even if rotation and perspective effects are carefully taken into account. Also the deviation of the observed CMPM from the true one depends on structural and kinematical properties (e.g.,  $V_c$ ) of the LMC. The present study however can not claim that the 3D motion of the LMC by K06a deviates significantly from the true one, but it strongly suggests that there is a possibility that the observed 3D motion using the PMs for a small number of the LMC fields *can* be significantly different from the true one owing to local random motions of stars in the LMC.

The possible deviation is very significant in terms of modeling the MS, because only a  $\sim 10 \text{ km s}^{-1}$  difference in a velocity component can cause a significant difference in the orbital evolution of the MCs (e.g., the longevity of the MCs’ binary status) that is crucial for the formation process of the MS (e.g., MF80). Our study also has pointed out that  $\sim 300$  fields of the LMC are necessary to derive a quite accurate CMPM for the LMC, because the effect of local random motion in deriving the CMPM can become negligible for such a large number of the fields. The present study suggests that the CMPM of the SMC derived by K06b can be less accurate than that of the LMC, firstly because only a small number of the SMC fields (5) are used for the PM measurement, and secondly because the stellar component can be better modeled as a system dynamically supported by velocity dispersion (e.g., Bekki & Chiba 2009).

The latest PM study of the LMC by Vieira et al. (2010, V10) has derived the CMPM by simply making an average of the observed PMs for 3822 stars and thereby has shown that the CMPM is different from those derived in previous studies (e.g., K06a,b; P08); V10 have also shown that the LMC is currently orbiting the Galaxy with  $V_{\text{LMC}} = 343 \pm 47.8 \text{ km s}^{-1}$ , which is significantly smaller than  $V_{\text{LMC}} \sim 380 \text{ km s}^{-1}$  derived by K06a using the *HST* proper motion results. Our results suggest that the simple average for 3822 stars in V10 can give an accurate CMPM, if observation-related errors for the PM measurement are really small for each field (star). This suggests that the current 3D velocity of the LMC by V10 should be also properly included in modeling the MCs, in particular, the formation of the MS.

If  $V_{\text{LMC}}$  by V10 is really close to the true one, then the results by V10 would have a profound implication for the dynamical modeling of the MS formation. Recent dynamical models that are consistent with the CMPM by K06a (i.e., higher  $V_{\text{LMC}}$ ) have failed to reproduce the bifurcated structure of the MS and the elongated leading arms self-consistently (e.g., Besla et al. 2010; R10). DB11 have successfully reproduced the above two fundamental

observations in a model with bound orbits of the MCs (with respect to the Galaxy) yet a rather high circular velocity ( $250 \text{ km s}^{-1}$ ) of the Galaxy. They thus suggested that bound orbits of the MCs are necessary to reproduce the two observations.

If the present 3D motion of the LMC by V10 is pretty close to the true one, then the LMC is bound to the Galaxy with the maximum circular velocity of  $220 - 250 \text{ km s}^{-1}$ . This means that the MS can be formed as a result of the LMC-SMC-Galaxy interaction, as shown by previous MS formation models with lower  $V_{\text{LMC}}$  (e.g., GN96). Given different values of the present 3D velocities of the LMC derived by different observations (e.g., K06a, P08, C09, and V10), future theoretical studies for the MS formation would need to investigate at least all of the three representative models with  $V_{\text{LMC}}$   $300 \text{ km s}^{-1}$  (C09),  $340 \text{ km s}^{-1}$  (V10), and  $380 \text{ km s}^{-1}$  (K06a). In this situation, it would be fair to claim that a theoretical MS formation model that can best reproduce fundamental properties of the MS and the leading arms would predict the LMC CPM that is closest to the true one.

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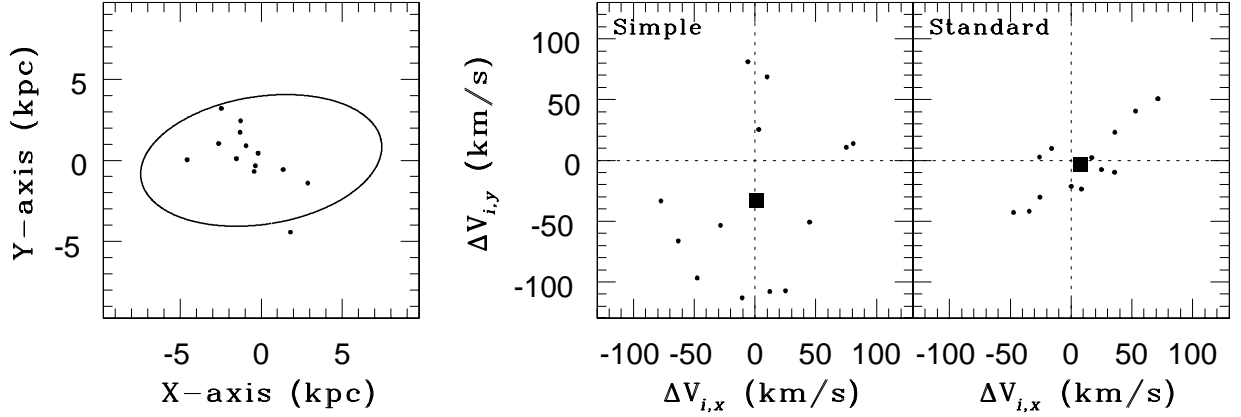


Fig. 1.— The left panel shows the spatial distribution of the selected 13 fields projected onto the  $x$ - $y$  plane in the fiducial LMC model with  $Q = 0$  and inclination angles  $\theta$  and  $\phi$  being  $45^\circ$  and  $30^\circ$ , respectively. The elongated circle describes the outer edge of the LMC disk ( $R_d = 7.5$  kpc). In the right two panels, each small dot describes the deviation of the simulated CM velocity of each field from the true one ( $\Delta \mathbf{V}_i = \mathbf{v}_{\text{cm},i} - \mathbf{V}_i$ ) derived by the simple method (left) and by the standard one (right). Here the  $x$ - and  $y$ -components of  $\Delta \mathbf{V}_i$  ( $\Delta V_{i,x}$  and  $\Delta V_{i,y}$ , respectively) are shown. The big filled square represents the average of  $\Delta \mathbf{V}_i$  and thus describes the deviation of the simulated CM velocity of the LMC from the true one. The simulated LMC CM velocity can deviate from the true one even in the model owing to the thick disk with non-zero vertical velocity dispersion.

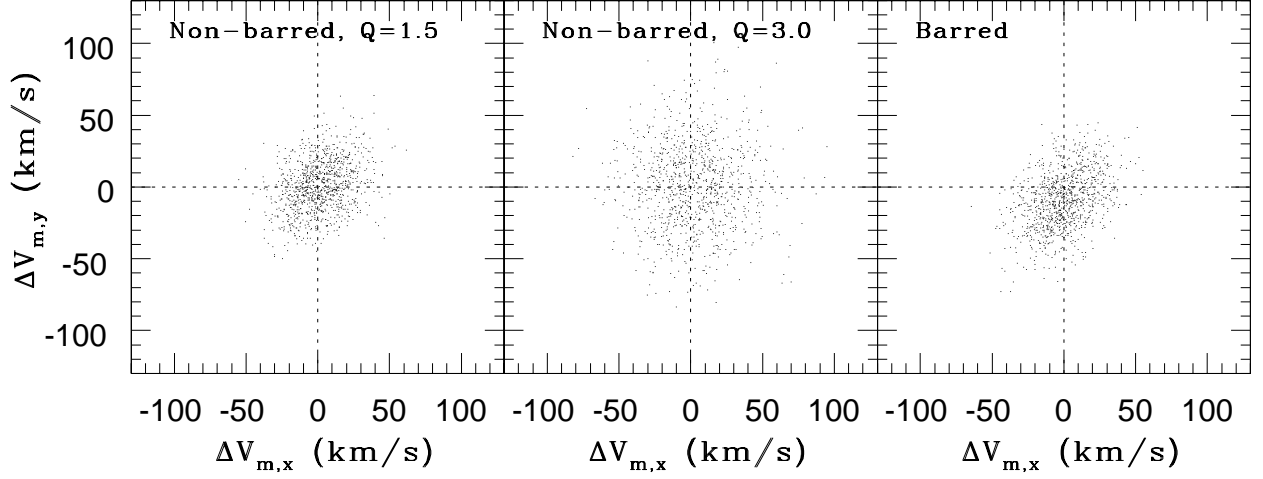


Fig. 2.— The plot of  $\Delta V_{m,y}$  as a function of  $\Delta V_{m,x}$  for each data sample in the fiducial LMC model with  $Q = 1.5$  and no bar (left),  $Q = 3.0$  and no bar (middle), and a central stellar bar (right). 10 fields are used in deriving  $\Delta \mathbf{V}_m$  for each sample and the results of 1000 samples are shown in each frame. It is clear that the simulated CM velocity of the LMC can significantly deviate from the true one owing to local random motion (i.e., non-zero velocity dispersion), in particular, in the model with a higher degree of random motion ( $Q = 3.0$ ).

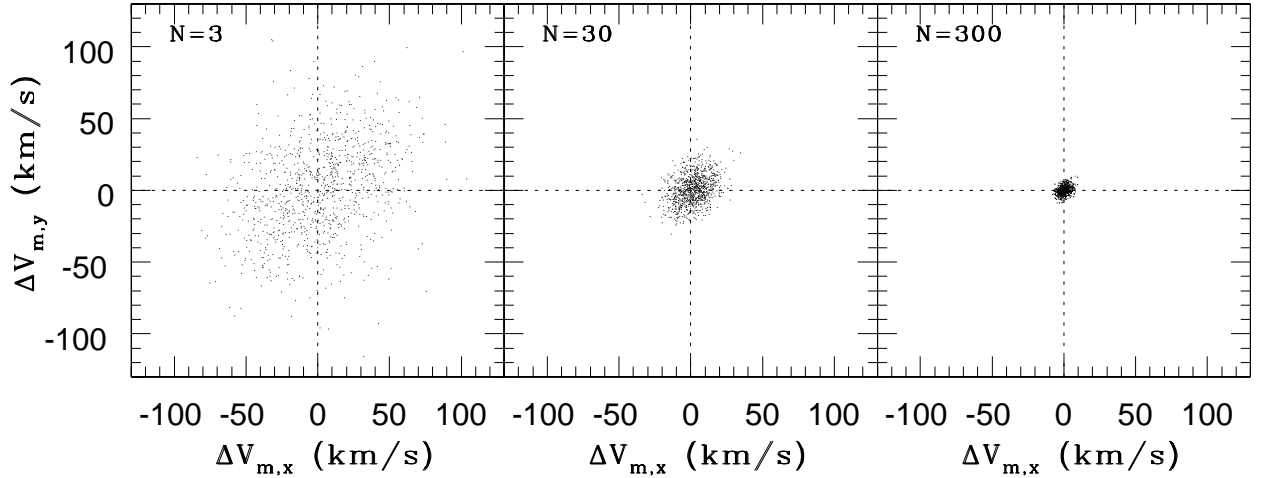


Fig. 3.— The same as Fig. 2 but for the fiducial LMC model with  $N = 3$  (left), 30 (middle), and 300 (right).

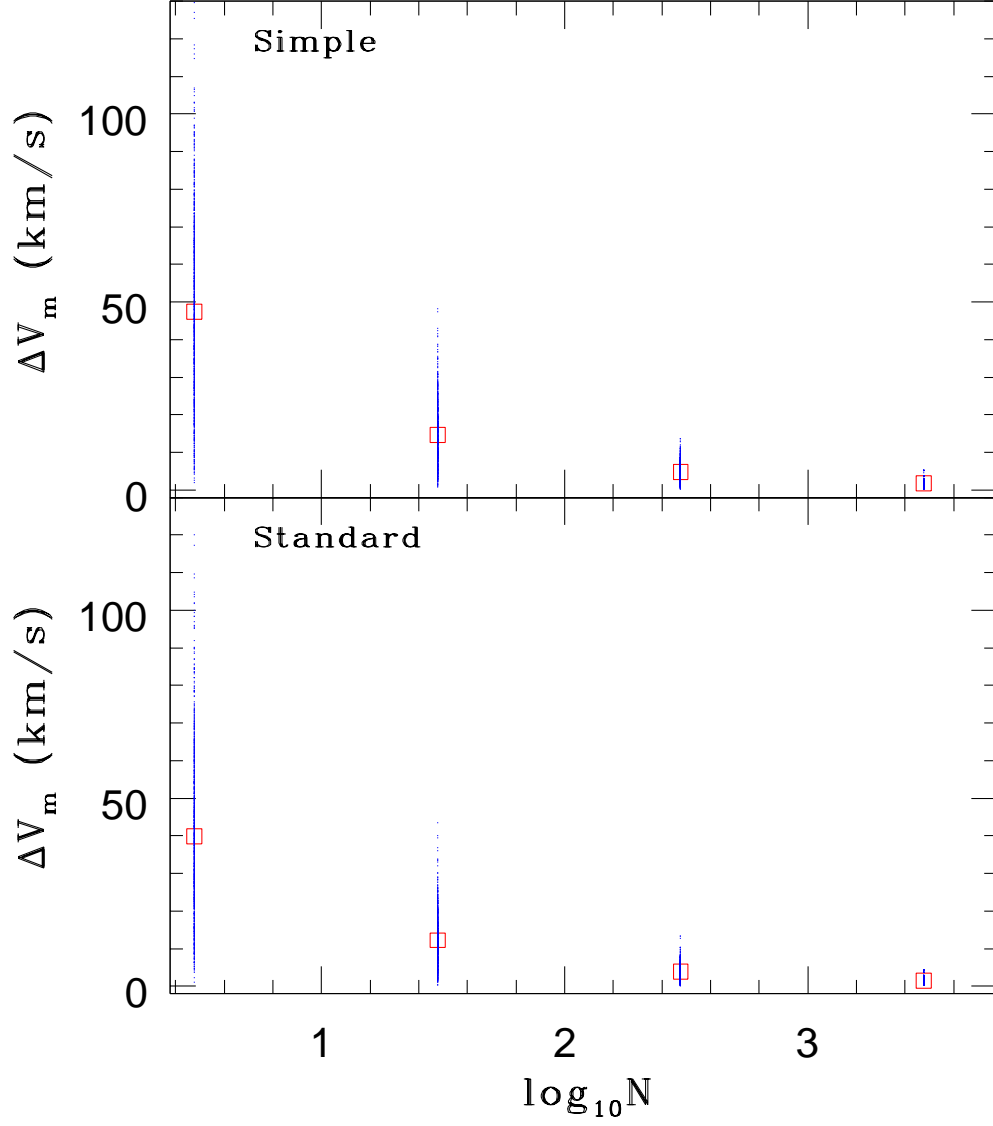


Fig. 4.— The distribution of  $\Delta V_m$  at each  $N$  ( $=3, 30, 300$ , and  $3000$ ) in the simple (upper) and standard methods (lower). Each blue dot represents the result of each sample. The red open squares represent the mean values of  $\Delta V_m$  for 1000 samples at each  $N$ .